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Inverse Extended Kumaraswamy Exponential Distribution with application to health data of Nepal

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ABSTRACT

This work is based on creation of a novel double-parametric probability model named Inverse Extended Kumaraswamy Exponential (IEKwE) Distribution taking the inverse of the random variable in Extended Kumaraswamy Exponential Distribution using the inverse transformation technique. In this study, we introduce expressions for various statistical functions as survival function, quantile function, and hazard function. We have also provided visual representations of the probability density and hazard rate curves. To evaluate the effectiveness and suitability of our formulated model, we have utilized a real-life dataset. We have estimated the model's parameters using three estimation tools.Validation of the model defined is tested using various statistical crteria and the goodness of fit is examined using graphical as well as the statistical inferential techniques. Additionally, we have used P-P and Q-Q plots for validation. We have empirically shown that, when compared to various alternative lifetime distributions, the suggested distribution offers a better fit and greater flexibility for lifetime data. All numerical and graphical measurements are conducted under the R programming software.Most important feature of this article is that it is two constant novel and have more flexibility compared to some models found in literature.

KEYWORDS
Extended Kumaraswamy Exponential Distribution, Maximum Likelihood Estimation, Model formulation, Goodness of fit, Quantile function, survival function

1. Introduction

Probability distributions are pivotal in applied sciences like medicine, engineering, and finance, as well as statistical modeling and analyzing lifetime data. Various lifetime distributions have been utilized for modeling this type of data. The realm of probability and statistics has seen ongoing advancement, with researchers discovering new distributions to represent real-world phenomena.There are various reasons behind formulating custom model. One of the important reason of formulation new probability model is to define model that fit the modern real dataset with better accuracy and validation compared to the classical models. Although the classical

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models defined in theory are base and ancestors of all the models which authors have been generating newels, but it gives better fit for some scenario compared to classical models. Custom models may have less number of outliers with satisfying maximum criteria of the probability models. Common methods for generalizing new distributions include distribution compounding, mixing, generations based on other models, power transformations, and inverted transformations. Over past few decades, researchers have increasingly focused on inverting univariate probability models and their use in inverse transformations of variables.

The Exponential distribution finds wide application across various fields due to its constant failure rate function. Contemporary statistical literature introduces modified extensions of the exponential distribution to address challenges. Many authors developed transformation of exponential model in the literature. Gupta and Kundu (1999; 2001) created the exponentiated exponential generalized exponential model. It's a subset of the exponentiated Weibull model (Mudholkar & Srivastava, 1993), sharing similarities in its right-skewed, uni-modal density function and monotone hazard function with gamma and Weibull distributions. These variations encompass a wide range of statistical models, each offering unique characteristics and applications. Illustrative instances encompass the beta exponential distribution (Nadarajah & Kotz, 2006) and beta generalized exponential (Barreto-Souza et al., 2010). Furthermore, the Kumaraswamy Weibull distribution by (Cordeiro et al., 2010) and the extension of the exponential distribution by (Nadarajah & Haghighi, 2011) enhance this varied landscape

New and unique probability distributions have emerged in recent years, expanding the landscape of statistical modeling. These innovative distributions encompass the Kumaraswamy exponential distribution introduced by (Cordeiro & de Castro, 2011), the gamma-exponentiated exponential distribution proposed by (Ristić & Balakrishnan, 2012), and the transmuted exponentiated distribution developed by (Merovci,2013). Additionally, researchers have introduced other intriguing options such as the extended exponential distribution (Gomez et al., 2014), the exponentiated exponential geometric distribution (Louzada et al., 2014), and the exponentiated generalized extended exponential distribution (de Andrade et al., 2016). These diverse distributions offer valuable tools for modeling various types of data and exploring new frontiers in statistical analysis.

Researchers have proposed various modifications, such as the modified exponential (Rasekhi et al., 2017), odd exponentiated half-logistic exponential (Afify et al., 2018), and Kumaraswamy extension exponential (Elbatal et al., 2018) distributions. Additionally, other options include the alpha power extended exponential (Hassan et al., 2018), Marshall-Olkin logistic-exponential alpha power exponential (Nassar et al., 2019), and extended odd Weibull exponential (Afify and Mohamed, 2020) distributions.

Continuing this trend are the logistic NHE (Chaudhary & Kumar, 2020), modified NHE (Chaudhary & Sapkota, 2021), and modified inverse NHE(Chaudhary et al., 2022). The landscape also encompasses the half Cauchy extended exponential(Chaudhary et al., 2022), half Cauchy exponential geometric(Telee et al., 2022), half Cauchy exponential extension(Telee et al.,2022) and inverse exponentiated odd Lomax exponential distribution (Chaudhary et al., 2022). Recently, the inverse exponential power distribution (Chaudhary et al., 2023), modified generalized exponential (Telee & Kumar, 2023) distributions and new Extended Kumaraswamy Exponential Distribution (Chaudhary et al., 2024) have enriched this field even further. These adaptations enrich the distribution's applicability and effectiveness in modeling diverse datasets.

To make the entire study systematic and well performed,following structure is employed to organize the various sections of this study: In section 2, we introduce the Inverse Extended Kumaraswamy Exponential (IEKwE) Distribution with properties. In third section, estimation process managing section 4 for furnishing model parameter estimates based on real data. Additionally,illustrations of the different criteria utilized to assess the fitting of model and conclusion in fifth section .

2. Model Formulations

Formulation of Inverse Extended Kumaraswamy Exponential (IEKwE) model taking the inverse of the random variable in Extended Kumaraswamy Exponential (EKwE) Distribution (Chaudhary et al., 2023) is mentioned here. Probability distribution function of EKwE distribution is given by eq. 2.1

$$F(x;\lambda,\beta)=\left[1-\{\exp\left(-\lambda y\right)\}^{\left(1-e^{-\lambda y}\right)}\right]^{\beta};y\geq 0,\lambda>0,\beta>0 \tag{2.1}$$

Defining a random variable x such that x y = 1, probability distribution (2.1) modified as

$$F(x;\lambda,\beta)=1-\left[1-\left\{\exp\left(-\frac{\lambda}{x}\right)\right\}^{\left(1-e^{\left(-\frac{\lambda}{x}\right)}\right)}\right]^{\beta};x>0,\lambda>0,\beta>0 \tag{2.2}$$

Corresponding density function is defined by (2.3)

$$f(x;\lambda,\beta)=\left(\frac{\lambda\beta}{x^2}\right)\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\left[1-\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\right]^{(\beta-1)}\left[1+\left(\frac{\lambda e^{-\lambda/x}}{x}\right)-e^{-\lambda/x}\right] \tag{2.3}$$

Some properties of model defined are also studied below.

2.1. Survival and Hazard function

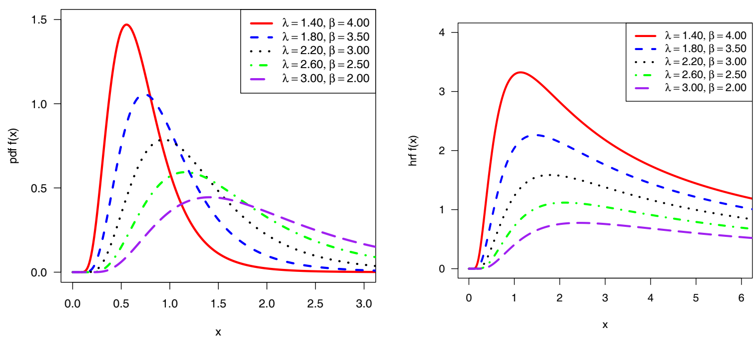
Survival function S(x) is the complementary function of the cdf and is the probability of surviving an event beyond x. The survival function of the proposed model is given in equation (2.4).

$$S(x)=\left(1-\left(e^{-\lambda/x}\right)^{\left(1-e^{-\lambda/x}\right)}\right)^{\beta};x>0,(\lambda,\beta)>0 \tag{2.4}$$

The h(x) of the proposed model is defined in equation (2.5).

$$h(x) = \left(\frac{\lambda\beta}{x^2}\right)\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\left[1-\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\right]^{-1}\left[1+\left(\frac{\lambda e^{-\lambda/x}}{x}\right)-e^{-\lambda/x}\right] \tag{2.5}$$

Figure 1 displays the probability density curve (left panel) for some values of the parameters. AS parameters varies, shape of the pdf curve varies indicating model can fit for different types of data sets. Right panel of the figure 1 contains hazard rate curves for some set of parameters. The shape of the hazard rate curve is increasing and decreasing type as well as the inverted bathtub shaped.The variations in shape of curves indicates the suitability of the model for fitting several types of new datasets.



(a) Probability density (b) Hazard rate function

Figure 1. Probability density curve and hazard rate curve.

2.2. Reversed and Cumulative hazard functions

The reversed hrex(x) and cumulative H(x) hazard functions of the model are defined in equation (2.6)and (2.7) respectively.

$$\begin{aligned} h_{rev}(x) = \left(\frac{\lambda\beta}{x^2}\right)\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\left[1-\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\right]^{\beta-1} \\ \left[1+\left(\frac{\lambda e^{-\lambda/x}}{x}\right)-e^{-\lambda/x}\right]\left[1-\left\{1-\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\right\}^{\beta}\right]^{-1} \end{aligned} \tag{2.6}$$

And,

$$H(x) \quad = \quad -\,lnS(x) \quad = \quad -\ \beta\ ln\left(1-\left(e^{-\lambda/x}\right)^{\left(1-e^{-\lambda/x}\right)}\right) \tag{2.7}$$

Additionally. equation (2.8) defines quantile function of IEKwE

$$x\log\left\{1-(1+p)^{(1/\beta)}\right\}+\lambda\left(1-e^{-\lambda/x}\right)=0; \quad p\in[0,1] \tag{2.8}$$

2.3. Asymptotic behavior

The density function's asymptotic behavior get verified if limiting values at zero and infinity have zero. If model satisfies the asymptotic properties, and then their modal value will exist. Taking limiting at end points,

$$\lim_{x\to 0}\left(\frac{\lambda\beta}{x^2}\right)\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\left[1-\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\right]^{(\beta-1)}\left[1+\left(\frac{\lambda e^{-\lambda/x}}{x}\right)-e^{-\lambda/x}\right]=0$$

$$\lim_{x\to \infty}\left(\frac{\lambda\beta}{x^2}\right)\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\left[1-\left\{e^{-\lambda/x}\right\}^{\left(1-e^{-\lambda/x}\right)}\right]^{(\beta-1)}\left[1+\left(\frac{\lambda e^{-\lambda/x}}{x}\right)-e^{-\lambda/x}\right]=0$$

Here, satisfying of asymptotic behavior shows that mode of model is defined. The variation in data is measured using quantile based Bowley's coefficient of skewness (Al-saiary et al., 2019) as,

$$\mathrm{SK\,(B)}\ =\frac{\mathrm{Q}\left[\frac{3}{4}\right]+\mathrm{Q}\left[\frac{1}{4}\right]\text{-}2*\mathrm{Q}\left[\frac{2}{4}\right]}{\mathrm{Q}\left[\frac{3}{4}\right]\text{-}\ \mathrm{Q}\left[\frac{1}{4}\right]}$$

Kurtosis formula by (Moors, 1988) is

$$\mathrm{K_u}=\frac{\mathrm{Q}\ \left[\frac{3}{8}\right]\text{-}\mathrm{Q}\left[\frac{5}{8}\right]\text{-}\mathrm{Q}\ \left[\frac{1}{8}\right]+\mathrm{Q}\ \left[\frac{7}{8}\right]}{\mathrm{Q}\ \left[\frac{6}{8}\right]\ \text{-}\ \mathrm{Q}\left[\frac{2}{8}\right]}$$

3. Methods for Estimation of model constants

Estimations of model's constants is crucial steps of any data analysis and modeling. It gives the b est values for the parameters that makes model suitable for fitting the data. The set of parameters for different data sets may be different to get better fitting. In this section we have discussed how to estimate the parameters and their important techniques. There are various methods available in literature for estimation of the parameters (constants) of the model. In this study, we have used three methods called maximum likelihood estimation, least square estimation method and Cramer – Von Mises method of estimation

3.1. Maximum Likelihood Estimation (MLE)

Let a random sample from IEKwE $\underset{-}{x} = (x_1, \ldots, x_n)$ where n is the size of sample. Equation (3.1) is the log likelihood function IEKwE

$$\ell(\lambda, \beta|\underline{x}) = n\log(\lambda\beta) - 2\sum_{i=1}^{n}\log x_i + (\beta-1)\sum_{i=1}^{n}\log\left[1-\left\{e^{-\lambda/x_i}\right\}^{\left(1-e^{-\lambda/x_i}\right)}\right]$$
$$-\lambda\sum_{i=1}^{n} x_i^{-1}\left(1-e^{-\lambda/x_i}\right) + \sum_{i=1}^{n}\log\left[1+\left(\frac{\lambda e^{-\lambda/x_i}}{x_i}\right) - e^{-\lambda/x_i}\right] \tag{3.1}$$

Beta and lambda can be estimated by solving partial derivatives of (3.1) with respect to λ, and β equated to zero. Solution of first order partial derivatives is not possible so computer programming can be used to solve the non-linear equations.

3.2. Method of Least-Square Estimation(LSE)

Let $\mathrm{X}_{(1)} < \mathrm{X}_{(2)} < \ldots < \mathrm{X}_{(\mathrm{n})}$ be ordered statistics and $\{X_i\}\,; i = 1, .., n$ is a of size n from a distribution function F (.). Let $F(X_{(i)})$ is CDF of ordered statistics by equation (3.2)

$$A\,(x; \lambda, \beta) = \sum_{i=1}^{n}\left[F(X_{(i)}) - \frac{i}{n+1}\right]^2 \tag{3.2}$$

Minimization of (3.2), with respect to the parameters and solving them, parameters of proposed model IEKwE can be obtained.

3.3. Method of Cramer-Von-Mises (CVM) estimation

To estimate λ, and β, equation (3.3) should be minimized

$$Z\,(X; \lambda, \beta) = \frac{1}{12n} + \sum_{i=1}^{n}\left[F\,(x_{i:n}|\lambda, \beta) - \frac{2i-1}{2n}\right]^2 \tag{3.3}$$

Differentiating (3.3) partially with respect to λ, and β, solving non linear equation estimated parameters can be obtained.

4. Real Data Analysis

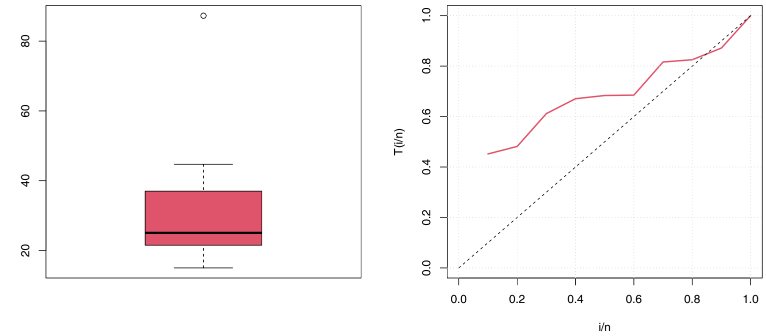
In this part of study, a real data set is introduced belonging to health data of Nepal. Data represents the leading causes of death per 100000 population of female in Nepal in 2019 [World Health Organization 2023 data.who.int, Nepal (Country overview) (Accessed on 4 September 2023)]. Data set: 87.3, 44.7, 37, 36, 25.1, 25, 24.3, 21.5, 16.1, 15.

Figure 2 reveals the TTT and Box-plot plots. Box plot shows that the data is positively skewed. The descriptive statistics of the dataset is mentioned in table 1. Descriptive

Table 1. Descriptive statistics of the data.

Min.	Q1	Q2	Q3	Mean	Max.	S.D.	Skewness	Kurtosis
15.00	22.20	25.05	36.75	33.20	87.30	21.2235	1.7828	5.362

statistics indicates that the data considered is positively skewed and non normal in nature.



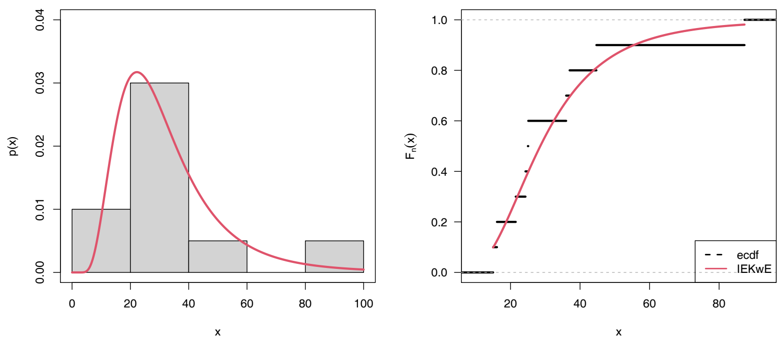
(a) Boxplot. (b) TTT plot.

Figure 2. Boxplot and TTT plot of the data.

Maximization of the likelihood function(3.1)using R software (R Core Team, 2023) with the optim() function to estimate the parameters by MLE method and are mentioned in table 2. Table 2 also contains the standard error of estimates of the parameters. Figure 3 displays the histogram and the fitted density plot of the model. Right panel of figure 3 displays the fitted cdf against the ecdf.

Table 2. MLE,LSE and CVM estimates of parameters.

Parameters	MLE	LSE	CVM
λ	50.7208	42.4222	51.5515
β	2.6685	1.9743	3.0408



(a) Histogram versus pdf (b) Empirical versus fitted cdf

Figure 3. Histogram versus pdf and Empirical versus fitted cdf

Table 3 compares the methods of estimation on the basis of four methods of information criteria values as well as negative values log-likelihood and finds that MLE method has the least information criteria values compared to other two methods. So, we conclude that it is better fit to the real data set compared to other methods

Table 3. Information criteria for different methods of estimation.

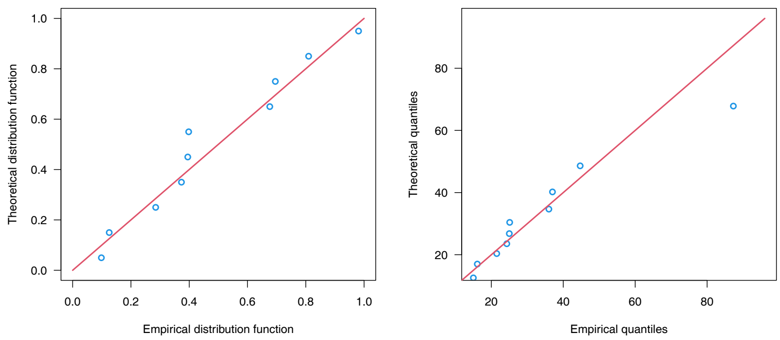
Methods	LL	AIC	BIC	CAIC	HQIC
MLE	-40.72189	85.44379	86.04896	87.15807	87.15807
CVM	-40.77818	85.55636	86.16153	87.27064	84.89249
LSE	-40.86634	85.73268	86.33786	87.44697	85.06881

Similarly, table 4 represents the test statistics and corresponding p values for all methods of estimation

Table 4. Goodness of fit statistics and p values for different methods of estimation.

Methods	KS(p-value)	W(p-value)	A^2(p-value)
MLE	0.2018(0.7396)	0.2932(0.9423)	0.0454(0.9137)
CVM	0.1637(0.9134)	0.0452(0.9145)	0.2978(0.9388)
LSE	0.1740(0.8737)	0.0391(0.9462)	0.2996(0.9374)

P-P plot and Q-Q plots are displayed in figure 4 showing that model fits better to the health data defined for study .



(a) P-P plot (b) Q-Q plot

Figure 4. P-P and Q-Q plot

4.1. Model Comparison

Model proposed in this study is compared to six other models found in literature. Six existing life time Models considered are Extended Kumaraswamy Exponential (EKwE) distribution (Chaudhary et al., 2023), Odd Lomax Exponential (OLE) distribution (Ogunsanya et al., 2019), Weibull Extension (WE) distribution (Tang et al., 2003), Generalized Exponential Extension (GEE) distribution (Lemonte,2013), Generalized Exponential (GE)distribution (Gupta& Kundu,1999) and Generalized Weibull Extension (GWE) (Sarhan and Apaloo, 2013). Table 5 contains the estimated parameters of all the models as well as the standard error of estimates for the give data set.

Information criteria values are tabulate in table 6. IEKwE has least information criteria values among all indicating that IEKwE fits the data set better.

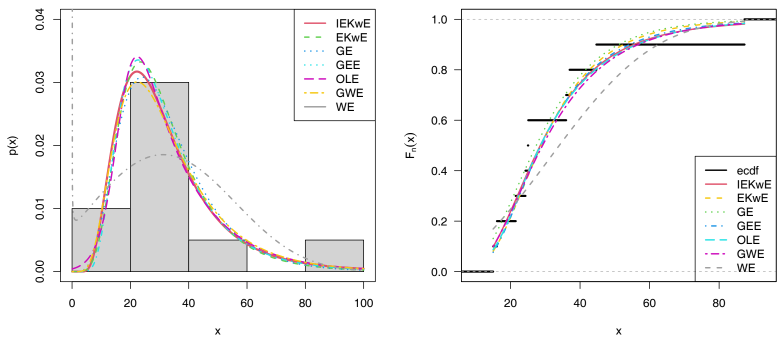
Table 5. Estimated parameters of competing models.

Methods	α	β	λ
IEKwE	50.7208(16.2783)	2.6685(1.6677)	-
EKwE	0.0687(0.0234)	3.4697(2.5619)	-
WE	0.0579(0.2236)	0.2879(0.0982)	0.0227(0.0244)
GEE	0.6197(0.1855)	17.5512(21.5016)	0.3241(0.3795)
OLE	0.1988(0.2227)	0.0072(0.0249)	3.4085(3.1046)
GWE	39.0241(62.6126)	0.2052(0.0270)	2.5321(3.6261)
GE	6.3923(4.2778)	-	0.0772(0.0246)

Table 6. Values relating information criteria for IEKwE as well as competing models.

Methods	LL	AIC	BIC	CAIC	HQIC
IEKwE	-40.72189	85.44379	86.04896	87.15807	87.15807
EKwE	-40.85428	85.70856	86.31373	87.42285	85.04469
GE	-41.12538	86.25076	86.85593	87.96504	85.58689
GEE	-40.60330	87.20661	88.11436	91.20661	91.20661
OLE	-40.72454	87.44909	88.35684	91.44909	86.45328
GWE	-40.84831	87.69662	88.60437	91.69662	86.70081
WE	-43.62468	93.24937	94.15712	97.24937	92.25356

Figure 5 displays the histogram versus fitted density function of all the competing distribution (Left panel) Right panel of the figure displays the empirical cdf versus fitted cdf of all the models. We conducted a comparison between the empirical dis-



(a) Histogram versus pdfs (b) Fitted cdf versus emperical cdfs

Figure 5. Histogram versus pdfs and fitted cdf versus emperical cdfs

tribution and the theoretical cumulative distribution of our proposed model. This comparison revealed that both curves closely align in the real data set we used for illustration. Additionally, we compared the theoretical cumulative distribution of our proposed model (referred to as IEKwE) with the theoretical cumulative distributions of EKwE, GE, GEE, OLE, GWE, and WE. Similarly, we compared the theoretical probability density function (PDF) of our model with those of other competing models. The results of our analysis says that our model has better fit for the given data set than all the other competitive models, as depicted in Figure 5. Table 7 represents the test statistics values as well as the corresponding p- values for all the three methods [the Kolmogorov-Smirnov (KS), Anderson-Darling (A2), and Cramer-Von Mises (W)] of testing the goodness of fit. The result verifies that the suggested model exhibits the minimum test statistic and a higher p-value. This suggests that the Inverse Extended Kumaraswamy Exponential (IEKwE) Distribution is the preferred choice when considering goodness-of-fit.

Table 7. Goodness of fit statistics and p values for different methods of estimation .

Methods	KS(p-value)	W(p-value)	A^2(p-value)
IEKwE	0.2018(0.7396)	0.2932(0.9423)	0.0454(0.9137)
EKwE	0.2183(0.6516)	0.3618(0.8832)	0.0504(0.8845)
GE	0.1679(0.8980)	0.0434(0.9242)	0.3852(0.8604)
GEE	0.2153(0.6680)	0.3229(0.9184)	0.0496(0.8893)
OLE	0.2061(0.7149)	0.3083(0.9305)	0.0473(0.9027)
GWE	0.2151(0.6689)	0.3423(0.9013)	0.0540(0.8626)
WE	0.2641(0.4161)	0.1642(0.3533)	0.9406(0.3874)

5. Conclusion

In this work, we have created a novel probability model named Inverse Extended Kumaraswamy Exponential (IEKwE) Distribution taking the inverse of the random variable in Extended Kumaraswamy Exponential Distribution using the inverse transformation technique. It is positively skewed and unimodal distribution. We've explored several statistical properties of the IEKwE model. This suggested model exhibits flexibility and encompasses both increasing and decreasing hazard functions, as well as an inverted bathtub-shaped hazard function, as revealed through graphical analysis of its Probability Density Function (PDF) and Hazard Rate Function (HRF). To assess the model's parameters, we employ the methods of Cramer's-von Mises Estimation (CVME), Least Squares Estimation (LSE), and Maximum Likelihood Estimation (MLE). Furthermore, by applying the IEKwE distribution to real- data set, we've observed its superior performance in terms of fitting compared to several other commonly used lifetime models. For both theoretical and applied statisticians, this model can serve as a valuable alternative for analyzing lifetime (survival) data or reliability data. Important feature of this study is that the model defined is novel and may be applicable for studying different data sets. The model defined here will help the researcher to know how to formulate the custom models as well as to use this model for analyzing the new data sets.

References

[1] Gupta, R. D. and Kundu, D. (2001). Exponentiated exponential family: an alternative to gamma and Weibull distributions. Biometrical Journal: *Journal of Mathematical Methods in Biosciences, 43(1), 117–130.* https://doi.org/10.1002/1521-4036(200102)43:1

[2] Mudholkar, G. S., and Srivastava, D. K. (1993). Exponentiated Weibull family for analyzing bathtub failure-rate data. *IEEE transactions on reliability,* 42(2), 299-302. https://doi.org/10.1109/24.229504

[3] Nadarajah, S. and Kotz, S. (2006). The beta exponential distribution. *Reliability engineering & system safety,* 91, 689–697.https://doi.org/10.1016/j.ress.2005.05.008

[4] Barreto-Souza, W., Santos, A. H. S. and Cordeiro, G. M. (2010). The beta generalized exponential distribution. *Journal of Statistical Computation and Simulation*, 80(2), 159–172.https://doi.org/10.1080/00949650802552402

[5] Cordeiro, G.M., Ortega, E.M.M. and Nadarajah, S. (2010). The Kumaraswamy Weibull distribution with application to failure data. *Journal of the Franklin Institute,* 347, 1399–1429.https://doi.org/10.1016/j.jfranklin.2010.06.010

[6] Nadarajah, S. and Haghighi, F. (2011). An extension of the exponential distribution. *Statistics, 45(6), 543–558.*https://doi.org/10.1080/02331881003678678

[7] Cordeiro, G. M., & de Castro, M. (2011). A new family of generalized dis-

tributions. *Journal of statistical computation and simulation, 81(7), 883-898.*https://doi.org/10.1080/00949650903530745

[8] Ristić, M. M. and Balakrishnan, N. (2012). The gamma-exponentiated exponential distribution. *Journal of Statistical Computation and Simulation, 82(8), 1191–1206.* https://doi.org/10.1080/00949655.2011.574633

[9] Merovci, F. (2013). Transmuted exponentiated exponential distribution. *Mathematical Sciences and Applications E-Notes, 1(2), 112–122.*

[10] Gomez, Y.M., Bolfarine, H. and Gomez, H.W. (2014). A new extension of the exponential distribution. *Revista Colombiana de Estadistica, 37(1), 25-34.*http://dx.doi.org/10.15446/rce.v37n1.44355

[11] Louzada, F., Marchi, V., & Roman, M. (2014). The exponentiated exponential–geometric distribution: a distribution with decreasing, increasing and unimodal failure rate. *Statistics, 48(1), 167-181.* https://doi.org/10.1080/02331888.2012.667103

[12] de Andrade, T. A. N., Bourguignon, M. and Cordeiro, G. M. (2016). The exponentiated generalized extended exponential distribution. *Journal of Data Science, 14(3), 393–413.* DOI=10.6339

[13] Rasekhi, M., Alizadeh, M., Altun, E., Hamedani, G., Afify, A. Z. and Ahmad, M. (2017). The modified exponential distribution with applications. *Pakistan Journal of Statistics, 33(5), 383–398.*

[14] Afify, A. Z., Zayed, M. and Ahsanullah, M. (2018). The extended exponential distribution and its applications. *Journal of Statistical Theory and Applications, 17(2), 213–229.*https://doi.org/10.2991/jsta.2018.17.2.3

[15] Elbatal, I., Louzada, F. and Granzotto, D. C. T. (2018). A new lifetime model: The Kumaraswamy extension exponential distribution. *Biostatistics and Bioinformatics,* 2, 1–9.http://dx.doi.org/10.31031/oabb.2018.02.000527

[16] Hassan, A. S., Mohamd, R. E., Elgarhy, M. and Fayomi, A. (2018). Alpha power transformed extended exponential distribution: properties and applications. *Journal of Nonlinear Sciences and Applications, 12(4), 62–67.*http://dx.doi.org/10.22436/jnsa.012.04.05

[17] Nassar, M., Kumar, D., Dey, S., Cordeiro, G. M. and Afify, A. Z. (2019). The Marshall–Olkin alpha power family of distributions with applications. *Journal of Computational and Applied Mathematics, 351, 41–53.*https://doi.org/10.1016/j.cam.2018.10.052

[18] Afify, A. Z. and Mohamed, O. A. (2020). A new three-parameter exponential distribution with variable shapes for the hazard rate: Estimation and applications. *Mathematics, 8(1), 135.* doi:10.3390/math8010135.https://doi.org/10.3390/math8010135

[19] Chaudhary, A. K., & Kumar, V. (2020). The Logistic NHE Distribution with Properties and Applications. *International Journal for Research in Applied Science & Engineering Technology (IJRASET), 8(12), 591-603.*http://dx.doi.org/10.22214/ijraset.2020.32565

[20] Chaudhary, A. K., & Sapkota, L. P. (2021). Modified NHE Distribution with Properties and Applications. *NUTA Journal, 8(1-2), 1-12.* https://doi.org/10.3126/nutaj.v8i1-2.44031

[21] Chaudhary, A. K., Sapkota, L. P., & Kumar, V. (2022). Modified Inverse NHE Distribution: Properties and Application. *Journal of Institute of Science and Technology, 27(1), 125-133. https://doi.org/10.3126/jist.v27i1.46695.*

[22] Chaudhary, A. K., Sapkota, L. P., & Kumar, V. (2022). Some properties and applications of half Cauchy extended exponential distribution. *Int J Stat Appl Math, 7(4), 226-235. https://doi.org/10.22271/maths.2022.v7.i4c.866.*

[23] Telee, L. B. S., Karki, M., & Kumar, V. (2022). Half Cauchy Exponential Geometric Distribution: Model, Properties and Application. *Interdisciplinary Journal of Management and Social Sciences,* 3(1), 96-110.http://dx.doi.org/10.3126/ijmss.v3i1.50236

[24] Telee, L. B. S., & Kumar, V. (2022). Some properties and applications of half-Cauchy exponential extension distribution. *International Journal of Statistics and Applied Mathematics,* 7(6): 91-101. https://doi.org/10.22271/maths.2022.v7.i6b.902

[25] Chaudhary, A. K., Telee, L. B. S., & Kumar, V. (2022). Inverse Exponentiated Odd Lomax Exponential Distribution: Properties and Applications. *Nepalese Journal of Statistics,*

6(01), 29-50. http://dx.doi.org/10.3126/njs.v6i01.50819

[26] Chaudhary, A. K., Sapkota, L. P., & Kumar, V (2023). Inverse Exponential Power distribution: Theory and Applications. *International Journal of Mathematics, Statistics and OperationsResearch,3(1)*,175-185. https://doi.org/10.47509/IJMSOR.2023.v03i01.11

[27] Telee, L. B. S., & Kumar, V. (2023). Modified Generalized Exponential Distribution. *Nepal Journal of Mathematical Sciences, 4(1)*.http://dx.doi.org/10.3126/njmathsci.v4i1.53154

[28] Chaudhary, A. K., Telee, L. B. S., Karki, M., & Kumar, V. (2024). Statistical analysis of air quality dataset of Kathmandu, Nepal, with a New Extended Kumaraswamy Exponential Distribution. Environmental Science and Pollution Research, 31(14), 21073-21088. https://doi.org/10.1007/s11356-024-32129-z

[29] Al-saiary, Z. A., Bakoban, R. A., & Al-zahrani, A. A. (2019). Characterizations of the Beta Kumaraswamy Exponential Distribution. *Mathematics, 8(1), 23*.https://doi.org/10.3390/math8010023

[30] Moors, J. J. A. (1988). A quantile alternative for kurtosis. Journal of the Royal Statistical Society: Series D *(The Statistician), 37 (1), 25-32*.https://doi.org/10.2307/2348376

[31] World Health Organization (2023 data). who.int, Nepal [Country overview]. (Accessed on 4 September 2023). Health data overview for the Federal Democratic Republic of Nepal. https://data.who.int/countries/524.

[32] R Core Team (2023). R: A Language and environment for statistical computing. (Version 4.1) [Computer software]. Retrieved from https://cran.r-project.org. (R packages retrieved from CRAN snapshot 2023-04-07).

[33] Chaudhary, A. K., Telee, L. B. S., & Kumar, V. (2023). Extended Kumaraswamy Exponential Distribution with Application to COVID-19 Data set. *Journal of Nepal Mathematical Society,* 6(1), 1-10.http://dx.doi.org/10.3126/jnms.v6i1.57657

[34] Ogunsanya, A. S., Sanni, O. O., & Yahya, W. B. (2019). Exploring some properties of odd Lomax-exponential distribution. *Annals of Statistical Theory and Applications (ASTA), 1, 21-30*.http://dx.doi.org/10.13140/RG.2.2.12002.27840

[35] Tang, Y., Xie, M., & Goh, T. N. (2003). Statistical analysis of a Weibull extension model. *Communications in Statistics-Theory and Methods, 32(5), 913-928*.https://doi.org/10.1081/STA-120019952

[36] Lemonte, A. J. (2013). A new exponential-type distribution with constant, decreasing, increasing, upside-down bathtub and bathtub-shaped failure rate function. *Computational Statistics & Data Analysis, 62, 149-170*.https://doi.org/10.1016/j.csda.2013.01.011

[37] Gupta, R. D. and Kundu, D. (1999). Generalized exponential distributions. *Australian & New Zealand Journal of Statistics, 41(2), 173–188*.https://doi.org/10.1111/1467-842X.00072

[38] Sarhan, A. M., & Apaloo, J. (2013). Exponentiated modified Weibull extension distribution. *Reliability Engineering & System Safety, 112, 137-144*.https://doi.org/10.1016/j.ress.2012.10.013

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